EXTRA PROBLEMS FOR HOMEWORK 7

1. Find the Fourier transforms of the following functions.

(a)
$$f(x) = e^{-|x|},$$
(b)
$$f(x) = \frac{1}{1+x^2},$$
(c)
$$f(x) = \begin{cases} 1, & |x| < 1, \\ 0, & |x| > 1, \end{cases}$$
(d)
$$f(x) = \frac{\sin x}{x}.$$

2. Find the solution of the heat equation on the line with the initial conditions from the previous problem.

3. Find the solution of the heat equation on the half-line with the following initial conditions and with Dirichlet and Neumann boundary conditions at $x = 0$.

(a)
$$f(x) = \begin{cases} 0, & 0 < x < a, \\ T_0, & a < x < b, \\ 0, & x > b, \end{cases}$$
(b)
$$f(x) = T_0 e^{-\alpha x}, \quad \alpha, x > 0.$$